### اللهم علمنا ما ينفعنا، وانفعنا بما علمتنا، وزدنا علما "سُبْحَانَكَ لا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيم"

# Ideal Low-Pass Filter Example

For the ideal low-pass filter shown in the image, let's go through an example using some numbers.  
In this example:  
- Let’s assume an image with size M x N = 8 x 8.  
- We’ll set a cutoff frequency D₀ = 3.

## Step 1: Calculate D (u, v)

For each frequency component at (u, v), D (u, v) is calculated as:  
D(u, v) = √[(u - M/2)² + (v - N/2)²]  
  
Since M = 8 and N = 8, the center is at (M/2, N/2) = (4, 4).

## Step 2: Calculate H (u, v) Using D₀

We’ll apply the transfer function based on

|  |  |  |
| --- | --- | --- |
| (u, v) | D(u, v) | H(u, v) |
| (4, 4) | 0 | 1 |
| (3, 4) | 1 | 1 |
| (4, 3) | 1 | 1 |
| (5, 4) | 1 | 1 |
| (4, 5) | 1 | 1 |
| (2, 4) | 2 | 1 |
| (4, 2) | 2 | 1 |
| (6, 4) | 2 | 1 |
| (4, 6) | 2 | 1 |
| (1, 4) | 3 | 1 |
| (4, 1) | 3 | 1 |
| (7, 4) | 3 | 1 |
| (4, 7) | 3 | 1 |
| (0, 4) | 4 | 0 |
| (4, 0) | 4 | 0 |
| (8, 4) | 4 | 0 |
| (4, 8) | 4 | 0 |

D (u, v): H (u, v) = 1 if D (u, v) ≤ D₀, 0 if D (u, v) > D₀  
  
this table shows how frequency components inside the radius D₀ = 3 pass through (with H (u, v) = 1), while those outside it are blocked (with H (u, v) = 0).

# Butterworth Low-pass Filter Example

## Example Parameters:

Filter Order n = 2  
Cutoff Frequency D₀ = 50  
Image/Signal Size: M = 256, N = 256 (so the center of the frequency plane is (M/2, N/2) = (128, 128))  
Frequency point: (u, v) = (150, 150)

## Step-by-Step Calculation:

### Step 1: Compute the Distance D(u, v):

Where D(u, v) = √((u - M/2)² + (v - N/2)²)  
Substitute the values:  
D(150, 150) = √((150 - 128)² + (150 - 128)²)  
D(150, 150) = √(22² + 22²) = √(484 + 484) = √968 ≈ 31.1

### Step 2: Calculate the Transfer Function H(u, v):

H(u, v) = 1 / [1 + (D(u, v) / D₀)²ⁿ]  
Substitute the values D(u, v) = 31.1, D₀ = 50, and n = 2:  
H(150, 150) = 1 / [1 + (31.1 / 50)⁴]  
31.1 / 50 ≈ 0.622, (0.622)⁴ ≈ 0.150  
H(150, 150) = 1 / [1 + 0.150] = 1 / 1.150 ≈ 0.87

# Butterworth Low-pass Filter Example2

Filter parameters: Cutoff frequency D0 = 50, Order n = 2  
Image size: 100x100

# Calculated Values at Example Points:

|  |  |  |  |
| --- | --- | --- | --- |
| u | v | D(u, v) | H(u, v) |
| 10 | 20 | 50.00 | 0.5000 |
| 50 | 50 | 0.00 | 1.0000 |
| 70 | 80 | 36.06 | 0.7872 |
| 90 | 90 | 56.57 | 0.3790 |

# Gaussian Low-pass Filter Example

## Transfer Function:

H(u, v) = e^(-D²(u, v) / 2D₀²)  
where D(u, v) = √((u - M/2)² + (v - N/2)²) is the Euclidean distance from the center.  
D₀ is the cutoff frequency.

## Example Parameters:

Cutoff Frequency D₀ = 50  
Image/Signal Size: M = 256, N = 256 (center of the frequency plane is at (128, 128))  
Frequency Points: (150, 150), (200, 200), (100, 100)

## Step-by-Step Calculations:

### Point 1: (u, v) = (150, 150)

Distance:  
D(150, 150) = √((150 - 128)² + (150 - 128)²) = √(484 + 484) = √968 ≈ 31.1  
Transfer Function:  
H(150, 150) = e^(-31.1² / (2 × 50²)) = e^(-968 / 5000) = e^(-0.1936) ≈ 0.824

### Point 2: (u, v) = (200, 200)

Distance:  
D(200, 200) = √((200 - 128)² + (200 - 128)²) = √(5184 + 5184) = √10368 ≈ 101.8  
Transfer Function:  
H(200, 200) = e^(-101.8² / (2 × 50²)) = e^(-10368 / 5000) = e^(-2.0736) ≈ 0.126

### Point 3: (u, v) = (100, 100)

Distance:  
D(100, 100) = √((100 - 128)² + (100 - 128)²) = √(784 + 784) = √1568 ≈ 39.6  
Transfer Function:  
H(100, 100) = e^(-39.6² / (2 × 50²)) = e^(-1568 / 5000) = e^(-0.3136) ≈ 0.731

## Summary of Results:

|  |  |  |
| --- | --- | --- |
| Point (u, v) | Distance D(u, v) | Transfer Function H(u, v) |
| (150, 150) | 31.1 | 0.824 |
| (200, 200) | 101.8 | 0.126 |
| (100, 100) | 39.6 | 0.731 |